Preference Learning based Decision Map Algebra: Specification and Implementation

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Abstract. Decision Map Algebra (DMA) is a generic and context independent algebra, especially devoted to spatial multicriteria modelling. The algebra defines a set of operations which formalises spatial multicriteria modelling and analysis. The main concept in DMA is decision map, which is a planar subdivision of the study area represented as a set of non-overlapping polygonal spatial units that are assigned, using a multicriteria classification model, into an ordered set of classes. Different methods can be used in the multicriteria classification step. In this paper, the multicriteria classification step relies on the Dominance-based Rough Set Approach (DRSA), which is a preference learning method that extends the classical rough set theory to multicriteria classification. The paper first introduces a preference learning based approach to decision map construction. Then it proposes a formal specification of DMA. Finally, it briefly presents an object oriented implementation of DMA.

Keywords: Decision Map Algebra, Preference learning, Dominance-based Rough Set Approach, Spatial modelling, Multicriteria modelling

1 Introduction

The GIS (Geographic Information System) is a powerful tool for collecting, storing, retrieving and analysing spatially-referenced data. Although GIS technology provides a large set of spatial analysis capabilities [10], it is still limited with respect to spatial multicriteria modeling where different decision alternatives and conflicting evaluation criteria and objectives need to be considered [7][23]. One possible solution to add spatial multicriteria modeling capabilities to the GIS is to develop a generic and context-independent language such as Tomlin's map algebra [36] or other similar tools such as image algebra developed by [30], the work of Serra on mathematical morphology [33], and the work of van Deursen and the PCRaster group on dynamic modeling [20][37].

Building on these pioneer works on map algebra, several new and domain-specific algebra have been proposed in the literature [2][5][15][32]. However none of initial and new map algebra are suitable to spatial multicriteria modelling.

To the best knowledge of the authors, the Decision Map Algebra (DMA) proposed in [6] is the first map algebra especially devoted to spatial multicriteria modeling within GIS technology. In this paper, we propose an object oriented implementation of the DMA within GIS technology, which constitutes the first step towards the development of a script-like spatial multicriteria modeling language. Furthermore, the development of abstract and generic framework for GIS-based multicriteria modeling is an important challenge as underlined in [3][21][26]

The rest of the paper is organized as follows. Section 2 introduces the decision map concept. Section 3 presents DMA. Section 4 reports on DMA modeling and implementation. Section 5 discusses some related work. Section 6 concludes the paper.

2 Preference Learning Based Decision Map

2.1 Definition and Construction of Decision Map

A decision map is a planar subdivision of the study area represented as a set of non-overlapping polygonal spatial units that are assigned, using a multicriteria classification model Γ, into an ordered set of classes or categories. More formally, a decision map $\mathbf{M}$ is defined as $\mathbf{M}=\{(u, \Gamma(u)) : u \in U\}$, where U is a set of homogenous spatial units and $\Gamma : U \to E$, where E is an ordinal measurement scale. The be useful, a decision map should be composed of non-overlapping spatial units.

The decision map construction procedure is composed of three main steps: (i) construction of criteria maps; (ii) overlay of these maps; and (iii) multicriteria Classification. The objective of the first step is to construct a set of criteria maps $c_1, c_2, \cdots, c_m$. The construction of criteria maps generally takes as input one or several basic maps. Then, a series of basic spatial operation (see [10]) are applied to combine these input maps into a new criterion map c defined such that each spatial unit in this map is characterized by a single evaluation $g(u)$ with respect to the criterion function g associated with the criterion map c.

The second step looks to overly the criterion maps $c_1, c_2, \cdots, c_m$, which leads to a multicriteria map composed of a new set of spatial units that result from the intersection of the boundaries of the features in the criteria maps. The multicriteria map may be described by the set $\{(u, g(u)) : u \in U\}$ with $g(u) = ((g_1(u), \cdots, g_m(u))$. This last vector represents the evaluations of spatial unit u with respect to evaluation criteria $g_1, g_2, \cdots, g_m$ associated with the criteria maps $c_1, c_2, \cdots, c_m$.

The aim of multicriteria classification is to apply a multicriteria classification model Γ on the multicriteria map obtained in terms of the previous step. The output is a decision map $\mathbf{M}=\{(u, \Gamma(u)) : u \in U\}$, where U is a set of homogenous spatial units.

We note that the criteria maps must represent the same territory and must be defined according to the same spatial scale and the same coordinate system. In addition, we mention that criteria maps must be polygonal ones. Non-polygonal input datasets can be easily transformed into polygonal ones using basic GIS analysis capabilities (see, e.g., [10][13]). It is also important to note that the overlay operation generally leads to a new set of spatial units resulting from the intersection of the boundaries of the spatial objects contained in the criteria maps. Finally, we note also that the overlay operation may generate silver polygons which should be eliminated.

Different methods can be used in the multicriteria classification step. In this paper, we advise to use a preference learning oriented method. The main argument beyond this proposition is to reduce the cognitive effort required from the experts and policymakers since they are not called to provide any preference parameter as with most of classical multicriteria classification methods. The principles of the preference learning method used in this paper are introduced in the rest of this section.

2.2 Principles of Preference Learning

Preference learning methods are primarily used to assess objects where decisions have been made and extract rules. Typically, this will be by taking an initial set of objects with known decisions (the learning set), applying an algorithm to extract rules and then applying these rules to predict the decision of new objects. The advantage of this approach are that the decisions can be predicted for the new objects without an extensive decision making process.

In this paper, we support the use of Dominance-based Rough Set Approach (DRSA) [16]. The DRSA is a preference learning method that extends classical rough set theory [28] to multicriteria classification. Rough set theory is a way of addressing analysis of imperfect data by taking lower (definitely belong) and upper approximations (possibly belong) of commonly held attributes between two objects. Figure 1 shows a rough set M, its lower approximation M_* and its upper approximation M^*. The set difference $Bn = M^* \setminus M_*$ between M^* and M_* is called the boundary.

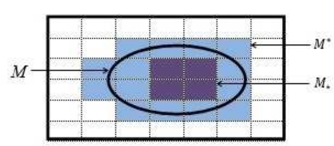


Fig. 1. Lower and Upper Approximations of Rough Set M [28]

In contrary to classical rough set theory and machine learning methods, DRSA assumes that attributes are preference-ordered. Preference-ordered attributes are commonly called criteria in multicriteria analysis. Furthermore, decision classes in DRSA are preference-ordered, while this is not a requirement in classical rough set theory and machine learning methods. In order to handle the monotonic dependency between criteria and decision classes, the DRSA uses two collections of union of classes defined as follows: (i) $Cl_t^{\geq} = \cup_{s \geq t} Cl_s$: upward union of classes; and (ii) $Cl_t^{\leq} = \cup_{s \leq t} Cl_s$: downward union of classes.

Then, DRSA determines the lower approximation for each union of classes (corresponding to objects, which according to their description certainly belong to the union of classes) and the upper approximation (corresponding to objects, which according to their description possibly belong to the union of classes). Boundary is the difference between lower and upper approximations. Therefore, rough set theory, especially DRSA, clearly separates certain and uncertain information [34]. Decision rules can then be extracted from the obtained approximations.

The input of DRSA is a learning dataset representing the description of a set of objects with respect to a set of criteria. The main output of DRSA is a collection of decision rules. A decision rule is a consequence relation $E \rightarrow H$ (read as If E, then H) where E is a condition (evidence or premise) and H is a conclusion (decision, hypothesis). Each elementary condition is built upon a single criterion while a consequence is defined based on a decision class. The obtained decision rules can then be applied to classify unseen objects. Figure 2 illustrates this working principle.

Fig. 2. Principles of DRSA

3 Decision Map Algebra

3.1 Specification of Decision Map Algebra

Symbols and Primitives To specify DMA, we adopt the classical algebraic specification method of [18][19]. This specification method consists of two parts: the syntactic specification and a set of axioms. For the purpose of an example, we define the types criterion_map and decision-map. A criterion_map is a mono-valued map layer where each spatial unit is characterized by one value representing the evaluation of this element with respect to a given criterion.

Type: **criterion_map**
set: `map_layer, spatial_unit, criterion_function, value`

A decision_map is a planar subdivision of the study area represented as a set of non-overlapping polygonal spatial units that are assigned into a set of preference-ordered classes through a multicriteria classifcation.

```
Type: decision-map
set:  multicriteria_map, criterion_map, learning_map, validation_map, decision_rules,
      spatial_unit, value
```

Syntax The first part of the specification defines the syntax for the operators of the data type. In the example below, we have three such operators that are associated with decision_map data type. The MAKE operator creates a decision map as the intersection of a set of criterion maps. The MERGE operator groups two or more adjacent spatial units of a given decision_map. The GROUP operator takes a decision_map as input and generates a new decision_map by merging all adjacent spatial units that are assigned to the same class.

```
MAKE  criterion_map ×···× criterion_map → decision_map
MERGE decision_map × spatial_unit × ··· × spatial_unit → spatial_unit
GROUP decision_map → decision_map
```

Axioms The third part of the specification is to define the behavior of the different operators. Following Guttag, there are two implicit axioms that must be present in any specification. The first set of axioms states that each operator returns "error" if any of its arguments does not belong to the domain of the operator. The second axiom states that an operator returns error if any of its arguments is "error".

The specifications of some operators relative to decision_map data type are given below. The MAKE operator uses the INTERSECT operator to combine a set of criteria_maps.

The MERGE operator groups two or more adjacent spatial units. It uses the MAKE operator, inherited from the basic polygon data type, to create a new spatial unit. The argument of MAKE is the boundary of the new spatial unit obtained by the union of the initial spatial units minus the common part (i.e. intersection of the boundaries of the initial spatial units). The evaluations of the new spatial unit with respect to all criteria are obtained by aggregating the evaluations associated with the initial spatial units. The operator ASSIGN of spatial_unit data type is used to assign the new evaluation to newly created spatial unit.

```
d: decision_map; u,u1,u2:spatial_unit; c1,···,cm,g: criterion_map;
r: decision_rules

MAKE(c1,···,cm)
= INTERSECT(c1,···,cm)

MERGE(d, u1, u2, f, op)
=u.make(d,  [BOUNDARIES(u1) ∪ BOUNDARIES(u2)]\
  [BOUNDARIES(INTERSECTION(u1, u2)]
  ∀(g)(g ∈ f)[ASSIGN(u,g,op.combine(SCORE(u1,g),SCORE(u2,g)))]

GROUP(d, op)
= ∀ (u1)(u2)(u1 ∈ d)(u2 ∈ d) ∧ (u1 <> u2)
  [if ADJACENT(u1, u2) u1.class = u2.class then
  MERGE(d, u1, u2, op, f)]
```

The specification of the MERGE operator is shown for two spatial units. The generalization to more than two spatial units is straightforward.

The GROUP operator takes as input a decision_map and generates a new decision_map by merging all adjacent spatial units that are assigned to the same class. The operator ADJACENT is used to test the adjacency of the spatial units in input. If the two spatial units are adjacent and have the same evaluation, then the operator MERGE is applied to merge them.

3.2 DMA Spatial Abstract Data Types

The spatial Abstract Data Types (ADT) supported by DMA are: criterion_map, weighted_criterion_map, multicriteria_map, learning_map, validation_map, decision_map andvalternatives_map. Each of these data types has a collection of proprieties and methods. Some of these methods permit to set or get the descriptive information of the corresponding data type while some others are devoted to set or get the spatial information of these data types. The criterion_map and decision_map data types have been introduced earlier.

The weighted_criterion_map is a specific version of criterion_map defined such that each spatial unit has a spatial weight. The multicriteria_map is obtained by overlying a set of $n > 1$ criteria maps or weighted criteria maps. Each spatial unit in the multicriteria_map is characterized by n scores corresponding to the n criteria maps in input.

The learning_map and validation_map are spatial representations of training and validation subsets often used in the application of preference learning methods. The training subset is labelled with known decisions and used to train the method. The validation subset is another subset of the input data with known decisions. We apply the preference learning method to this subset to see how accurately it identifies the known decisions.

The decision_map can used as it is to support suitability analysis as in [25]. In more complex decision problems, the use of a decision_map requires the definition of appropriate tools to generate solutions to the considered decision problem, which will lead to one or more alternatives_maps. Some formal solutions to generate alternatives_map have been proposed and used in [1][8][9]. These solutions rely largely on graph theory algorithms.

In the rest of this section, we present a summary of some properties and methods associated with criterion_map data type. The basic properties associated with a criterion_map are:

- NAME: a name that uniquely identifies the criterion_map,
- DESCRIPTION: a textual description of the criterion_map,
- DATA TYPE: the type of data represented in the criterion_map. It may be nominal, symbolic, ordinal, integer or continuous.
- POSSIBLE DATA VALUES: For nominal and symbolic data types, we need also to indicate the set of possible values.
- SCORE: the score of a spatial unit of the criterion_map.
- WEIGHT: the weight of the criterion_map.

- PREFERENCE: it indicates the direction of preference. Three cases are possible: (i) gain: an increase on the criterion value, will lead to a higher attractiveness; (ii) cost: an increase on the criterion value, will lead to a lower attractiveness; or (iii) none: this is for nominal or symbolic criteria with no preference structure.
- REFERENCE: it represents the geographic coordinate system used.
- MAP SCALE: it is the map scale, which is the ratio of a distance on the map to the corresponding distance on the ground.

The basic methods associated with a criterion_map data type are given in Table 1. A criterion_map supports two basic methods, namely SET and GET, permitting to set or get the value of any property. Each of these methods has two different syntaxes. The first one applies to all the above cited-properties, except SCORE for which a specific syntax for accessing the score of a spatial unit is used. The STANDARDIZE method permits to process the initial data represented by the criterion_map into form appropriate to multicriteria modelling. This method requires the specification of a standardization techniques.

Table 1. Some Methods Associated with criterion_map Data Type

Name	Syntax
SET	criterion_map × property × value → criterion_map
	criterion_map × spatial_unit × value → spatial_unit
GET	criterion_map × property → value
	criterion_map × spatial_unit → value
STANDARDIZE	criterion_map × std_procedure → criterion_map

4 Object Oriented Modeling and Implementation

In this section, we report an object oriented modeling of DMA. The adoption of an object oriented modeling formalism relies on the following reasons: (i) recent works on spatial data models are more and more oriented towards the object oriented modelling [12][22][31][32]; (ii) this approach seems to be in accordance with human perception of geographic space, often seen as "populated" with objects [11]; (iii) an object formalism is more adequate for developing and implementing abstract data types devoted to spatial multicriteria modelling.

Intuitively, each data type in DMA is defined as a class and the operators associated with this data type are defined as methods for this class. A simple version of the UML model associated with DMA is given in Figure 3. In addition to classes implementing the spatial ADT introduced earlier, the UML model contains a new class called Classifier which represents a set of rules.

Objects of Classifier class are the result of calling the function Infer on learning_map object. The classify method of Classifier class takes as input a learning_map and a validation_map and generates a decision_map. Classifier class contains several other basic methods:

(1) *lhs* and *rhs* that are used to access the left-hand-sides or right-hand-sides of the rules.
(2) a set of methods permitting to access different performance measures of decision rules such as support, confidence and strength.
(3) *covers* which permits to check if a given rule covers a given spatial unit.
(4) *supports* which is used to check if a a given spatial unit supports a given rule.

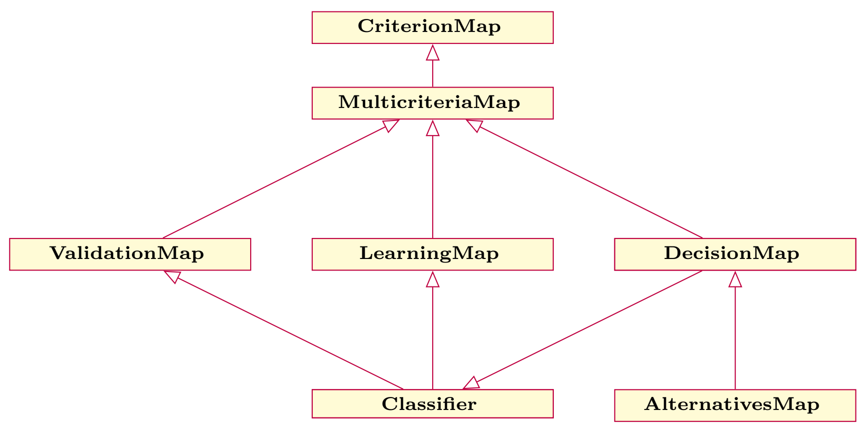
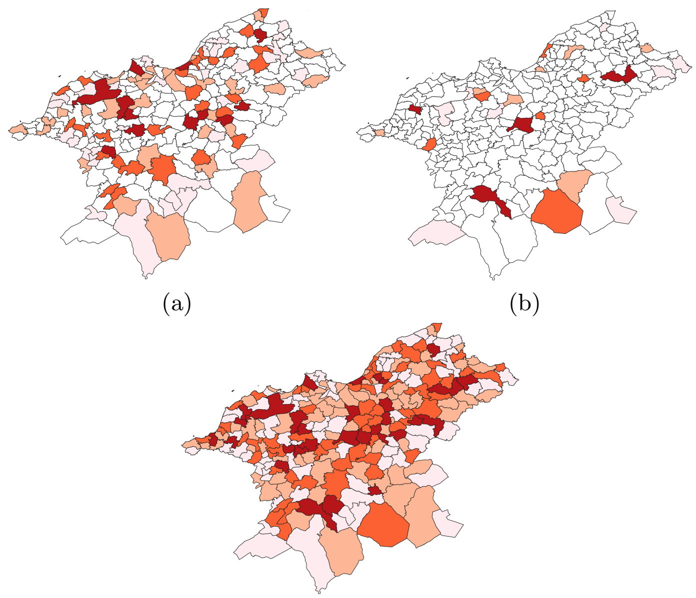


Fig. 3. UML Model

The DMA is being implemented using Python and QGIS platform. For illustration, we provide in Figure 4 three maps concerning seasonal influenza risk assessment in the northwest region of Algeria. The learning and validation maps are given in Figures 4(a) and 4(b), respectively, while the final risk map is shown in Figure 4(c). A four level risk scale ranging from 'Low' (light grey) to 'Very High' (dark red) has been used in the three maps. The assignment of the districts in learning and validation maps have been specified by the experts while the assignments of the districts in the final risk map have computed using method classify of Classifier class.

5 Related Work

The Map Analysis Package (MAP) [36] was the first comprehensive collection of analytical and spatial operations on the basis of regular tessellations. MAP has been extended in area ranging from cellular automata [35], to environmental modeling [29], to topographic analysis [4], to spatio-temporal analysis [24]. An important limitation of MAP is related to the fact that it describes map overlay operations textually, without applying the mathematical rigor necessary to



(a) (b) (c)

Fig. 4. Illustrative Example

analyze the behavior of the operations. A second important problem of MAP is the strong link between geographical datatypes and data structures.

Another proposal that have inspired DMA is the one proposed [14]. A major finding of this paper is the use of decision table concept, which is adopted in DMA. More recent works are essentially devoted to develop script-like programming languages [29], to support spatial-temporal analysis [24], visual spatial modeling [27] and to the development of web-based map algebra-like frameworks [17].

There are also several new and domain-specific algebra that have been proposed in the literature [2][5][15][32]. For instance, authors in [32] propose an integrated modelling framework that provides descriptive means to specify (1) model components with conventional map algebra, and (2) interactions between model components with model algebra. A prototype implementation in a high-level scripting language supports the building of integrated spatio-temporal models is also proposed.

The authors in [5] design and implement a framework that uses compiler techniques to automatically speed up raster spatial analysis. In this way, users simply write sequential map algebra scripts in Python, which are translated into a graph where optimizations are applied.

In [2], the authors presents an algebra that extends the Systems Dynamics paradigm to the development of spatially explicit models of continuous change. The proposed algebra provides types and operators to represent flows of energy and matter between heterogeneous regions of geographic space. To this end, algebraic sets of operations similar to those in Map Algebras are introduced, allowing the representation of local, focal and zonal flows.

6 Conclusion

The paper provides an object oriented modelling and implementation of Decision Map Algebra (DMA). This is constitutes the first step towards the development of a script-like spatial multicriteria modeling language. From theoretical point of view, the paper mainly enhances DMA through the use of preference learning based approach to decision map construction. This will naturally reduce the cognitive effort required from the experts and policymakers since they are not called to provide any preference parameter.

Our current research concerns the full implementation of the proposed data types. We also intend to design and implement a script language for spatial multicriteria modeling. We are also concerned by the design and development of a graphical version of the script language.

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